Redundancy conjecture and super-capacity rate of increase

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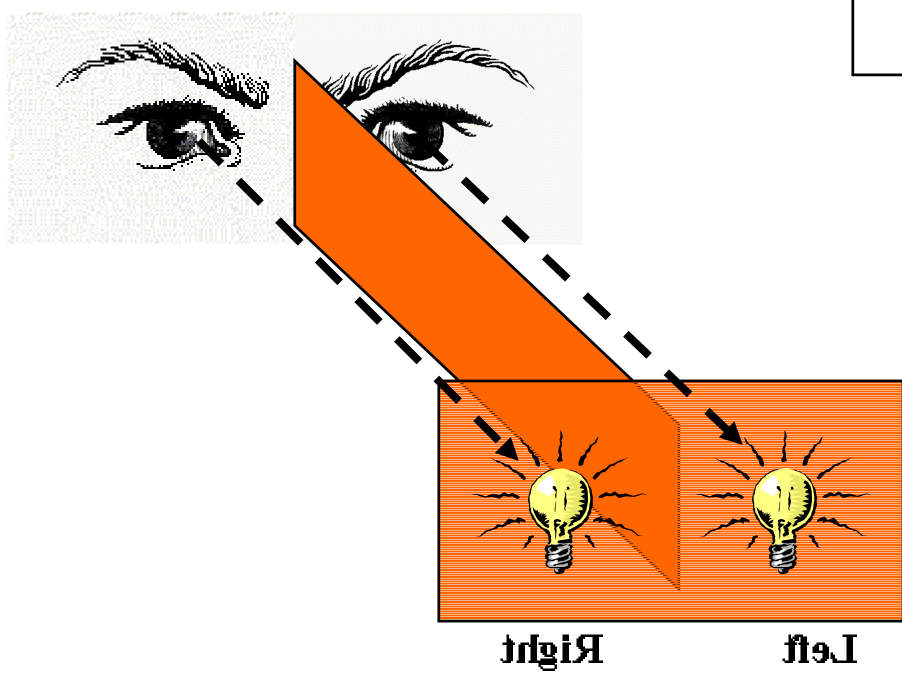
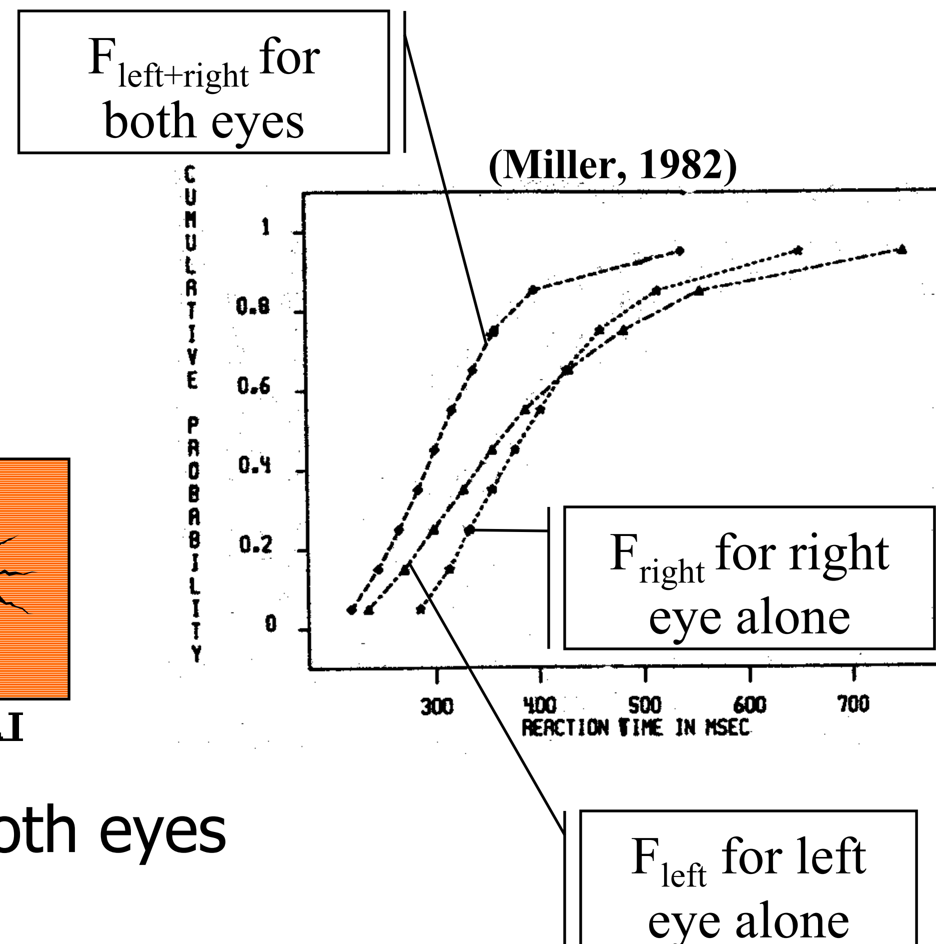
Redundant signals:
a) The basic paradigm

How do we respond in the presence of a signal in one eye, both eyes, or in none?

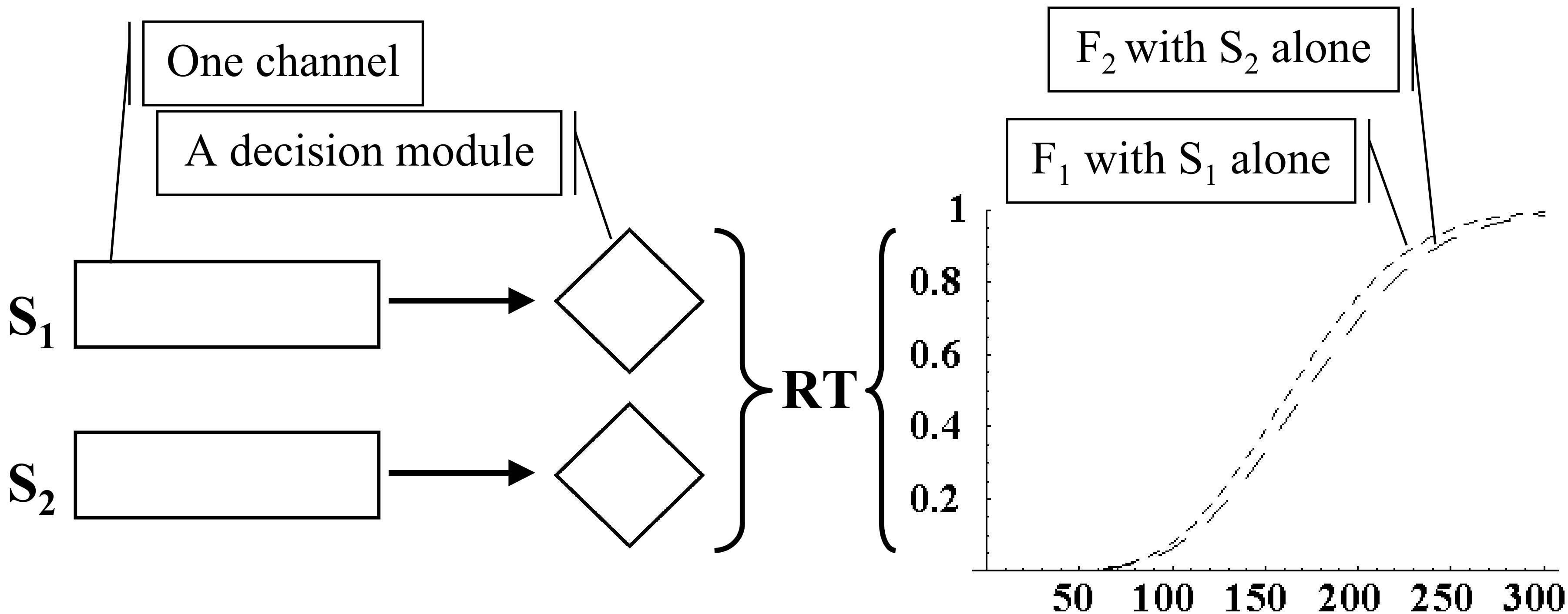




How good are the results of both eyes compared to a single eye?

Redundant signals:
b) The basic model

A parallel independent-channel self-terminating decision model:



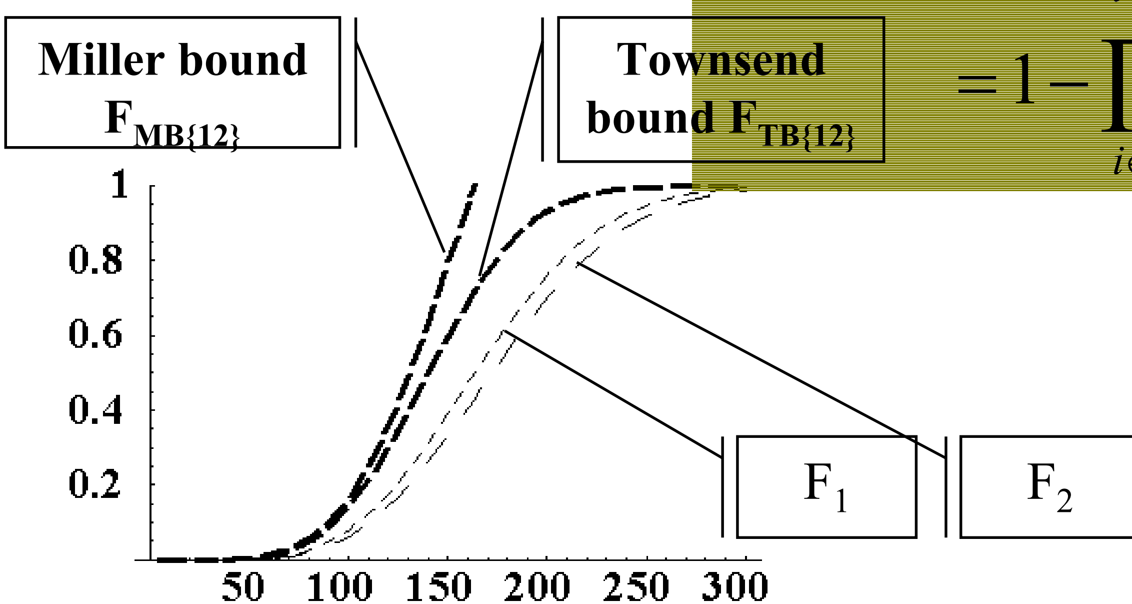
When both signals are presented, statistical facilitation should occur, speeding up the $F_{\{12\}}$ RTs.

Redundant signals:
c) The theoretical bounds

A bound on the amount of statistical facilitation can be computed:

$$F_{TB\{S\}}(t) = \Pr\{Min(\mathbf{T}_i) \le t\}$$
$$= 1 - \Pr\{all\ \mathbf{T}_i > t\}$$
$$= 1 - \prod_{i \in S} \{1 - F_i(t)\}$$
$$= 1 - \prod_{i \in S} S_i(t)$$

This formulae is valid for more than two channels.



Where $S_i(t) := 1 - F_i(t)$ is the survivor function.

Redundant signals:
c) Capacity measure as deviation

The Townsend bound indicates a lower limit to the best performance expected from the basic model.

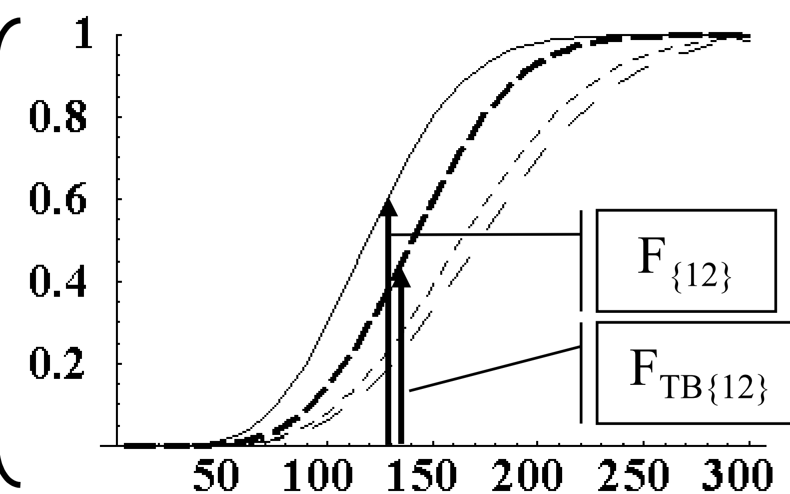
If there is a deviation, we need to quantify it at times t:

$$C_{\{S\}}(t) := \frac{\log S_{\{S\}}(t)}{\sum_{i \in S} \log S_i(t)} = \frac{\log S_{\{S\}}(t)}{\log S_{TB\{S\}}(t)}$$

This formulae is also valid for more than two channels.

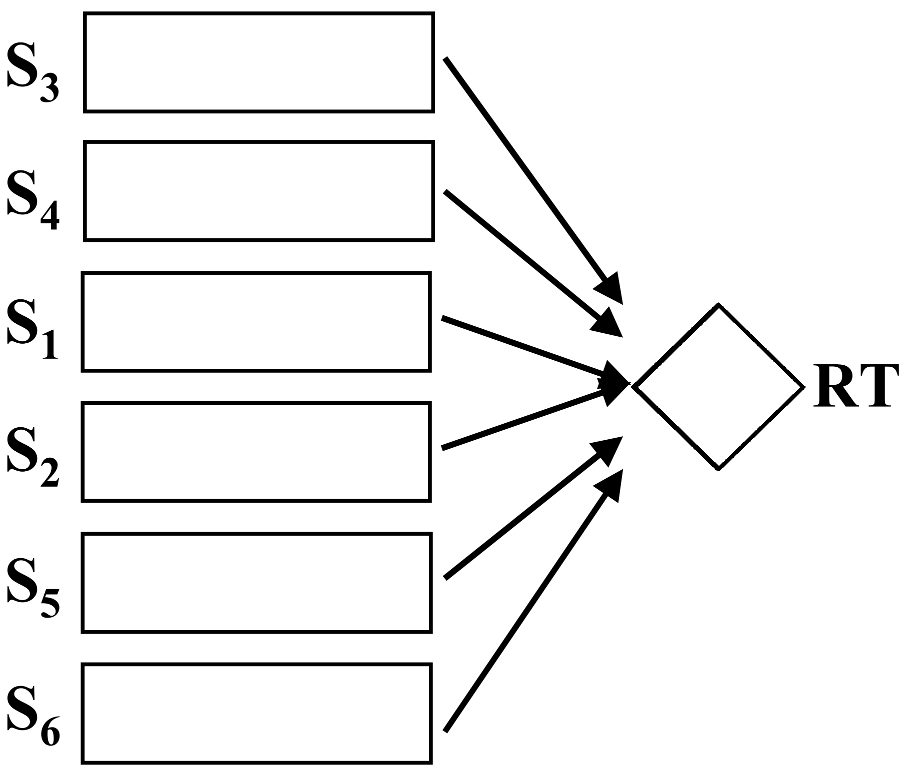
If $C_{\{S\}}(t) = 1$ for all t, then no deviation;

If $C_{\{S\}}(t) > 1$ for all t, then better-than-parallel performances i.e. "super-capacity" or coactivation.



An alternative model:
a) What is coactivation?

Coactivation is obtained by pooling all the evidences in the same decision mechanism.



Why be limited by only two channels?

As the number of inputs *#S* increases, better tests may be obtained.

An alternative model:
b) Incremental measure of capacity

For all subset G of the stimulus set S, we can compute a partial capacity measures $C_{\{G\}}$ where:

$$1 = \begin{Bmatrix} C_{\{1\}}(t) \\ C_{\{2\}}(t) \\ C_{\{3\}}(t) \end{Bmatrix} < \begin{Bmatrix} C_{\{1,2\}}(t) \\ C_{\{1,3\}}(t) \\ C_{\{2,3\}}(t) \end{Bmatrix} < C_{\{1,2,3\}}(t)$$

Proven in Townsend and Nozawa, 1995.

Proof trivial

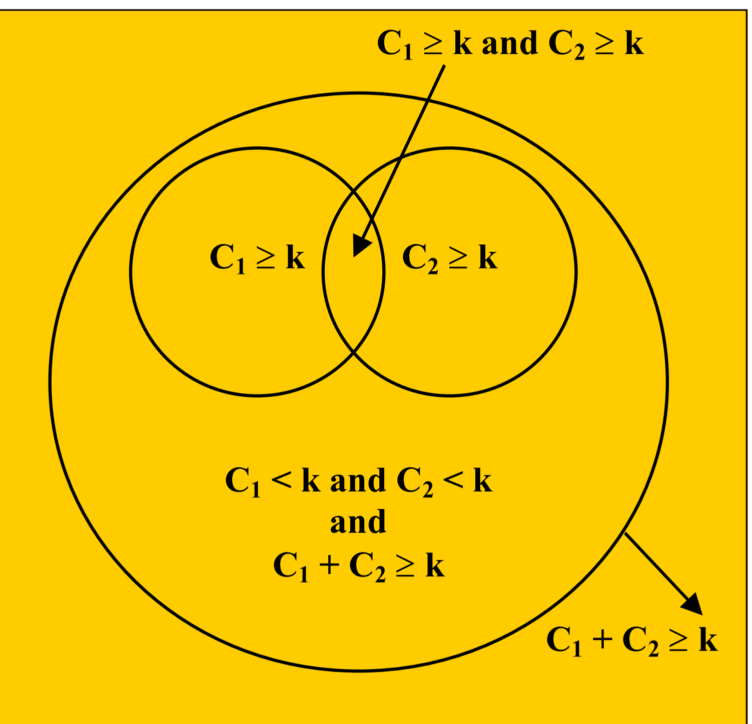
An alternative model:
c) Can we analyze $C_{\{S\}}(t)$?

Here, I assume only two channels acting as counters C_1 and C_2 with fixed criterion k.

$$F_{\{12\}}(t) = \Pr\{\mathbf{T}_{Sum(C_i)\geq k} \leq t) = \Pr\{\mathbf{T}_{C_1 \geq k} \leq t) + \Pr\{\mathbf{T}_{C_2 \geq k} \leq t)$$

$$- \Pr\{\mathbf{T}_{C_1 \geq k \wedge C_2 \geq k} \leq t)$$

$$+ \sum_{n_1=1}^{k-1} \sum_{n_2=k-n_1}^{k-1} \Pr\{\mathbf{T}_{C_1 \geq n_1} \leq t \wedge \mathbf{T}_{C_2 \geq n_2} \leq t)$$



Solution: no, because the last term has dependencies between the counters.

So far…

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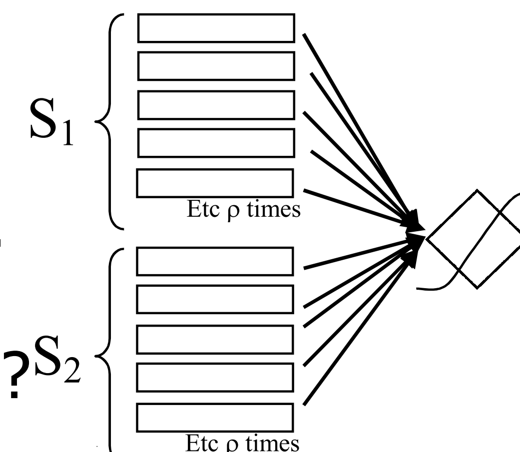
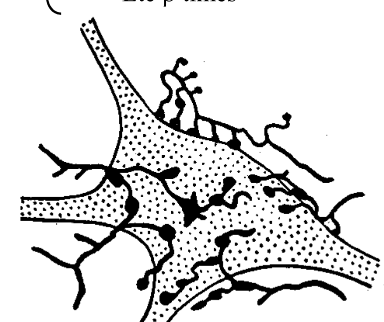
- We provided a generalized definition of $F_{\{S\}}$ and $C_{\{S\}}$ to more than two channels;
- And an incremental measure of capacity such that $C_{\{G\}} < C_{\{S\}}$ when $G \subset S$;
- Capacity cannot be solved analytically if the coactivation hypothesis is the only one present.



Why consider $\#S > 2$, we only have two eyes?

- The PRM is a counter model that assumes redundant input even within a channel:
  - Brain connections involve redundant pathways.
  - The PRM is analytical under mostly any conditions.
  - With redundancy, PRM can learn in much the same way neural networks do.



From Hebb, 1949.

Assuming PRM: Can I solve for $C_{\{S\}}(t)$? $= \dfrac{\log S_{\{S\}}(t)}{\sum_{i \in S} \log S_i(t)}$

1. Solving the denominator: Single-stimulus distributions

   $F_i(t) = \Pr\{\mathbf{T}_{C_i \geq k} \leq t\}$ exists iif $\Pr\{\mathbf{T}_{C_i \geq 1} \leq t\}$ exists.

   _k_, the accumulator size, is a free parameter.

   let $L_i(t) := \Pr\{\mathbf{T}_{C_i \geq 1} \leq t\} = W(\gamma, b_i)(t)$

   then $F_i(t) = 1 - (1 - L(t)) \sum_{j=0}^{k-1} \frac{1}{j!} Log\left(\frac{1}{1 - L_i(t)}\right)^j$ (Galambos, 1978)

2. Solving the numerator: Expected distribution in the {S} redundant-stimulus condition

   $F_{\{S\}}(t)$ is the k[th] fastest of #S pools of ρ redundant racers.

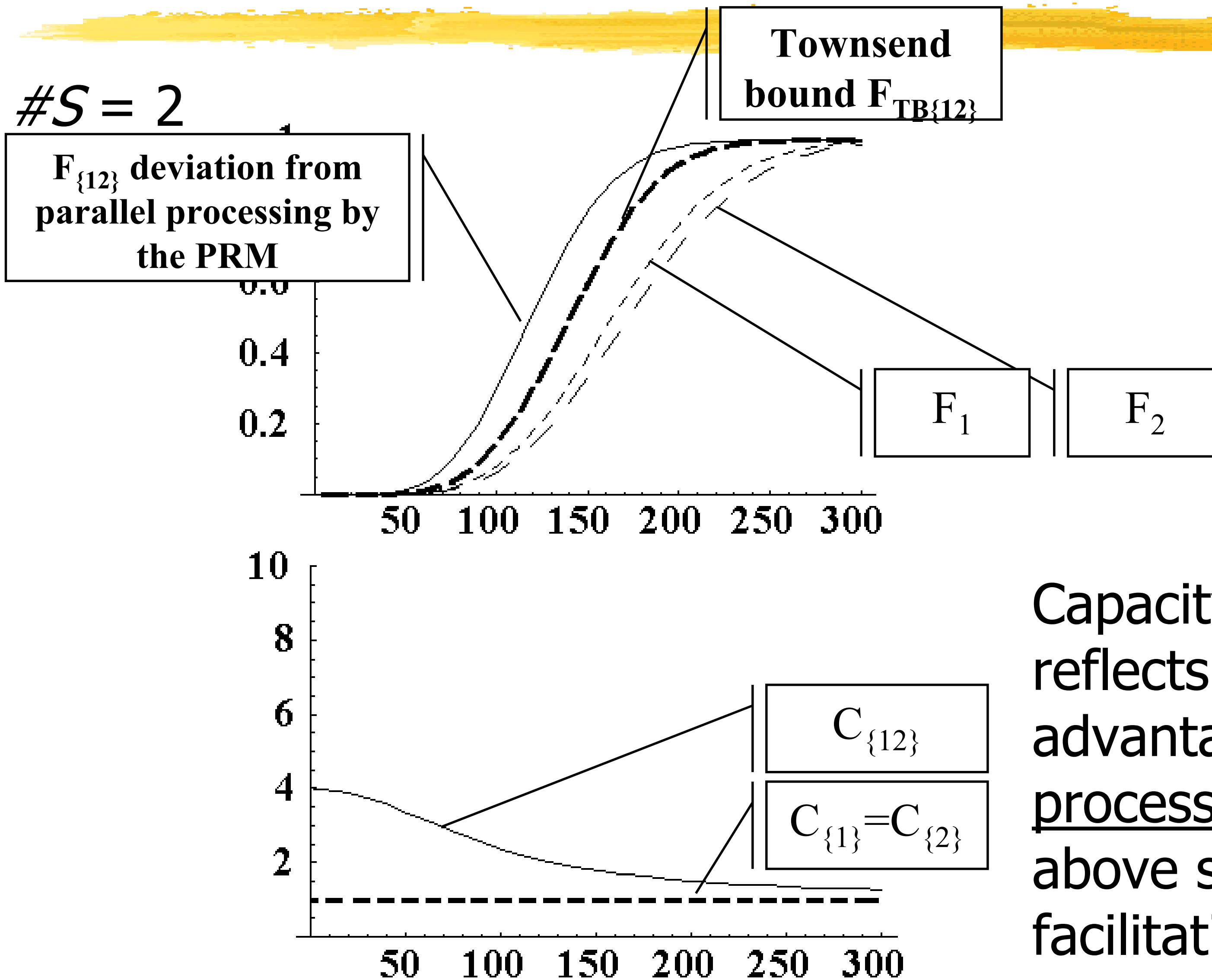
   Was previously unsolvable, but! If we pool all the racers together,

   $F_{\{S\}}(t)$ is the k[th] fastest of a single pool of #S $\times\ \rho$ racers, thus

   $L_{\{S\}}(t) = \Pr\{\mathbf{T}_{\underset{i \in S}{Sum}(C_i) \geq 1} \leq t) = W(\gamma, b_i / \sqrt[\gamma]{\rho})(t)$

Assuming PRM: Illustrating all this

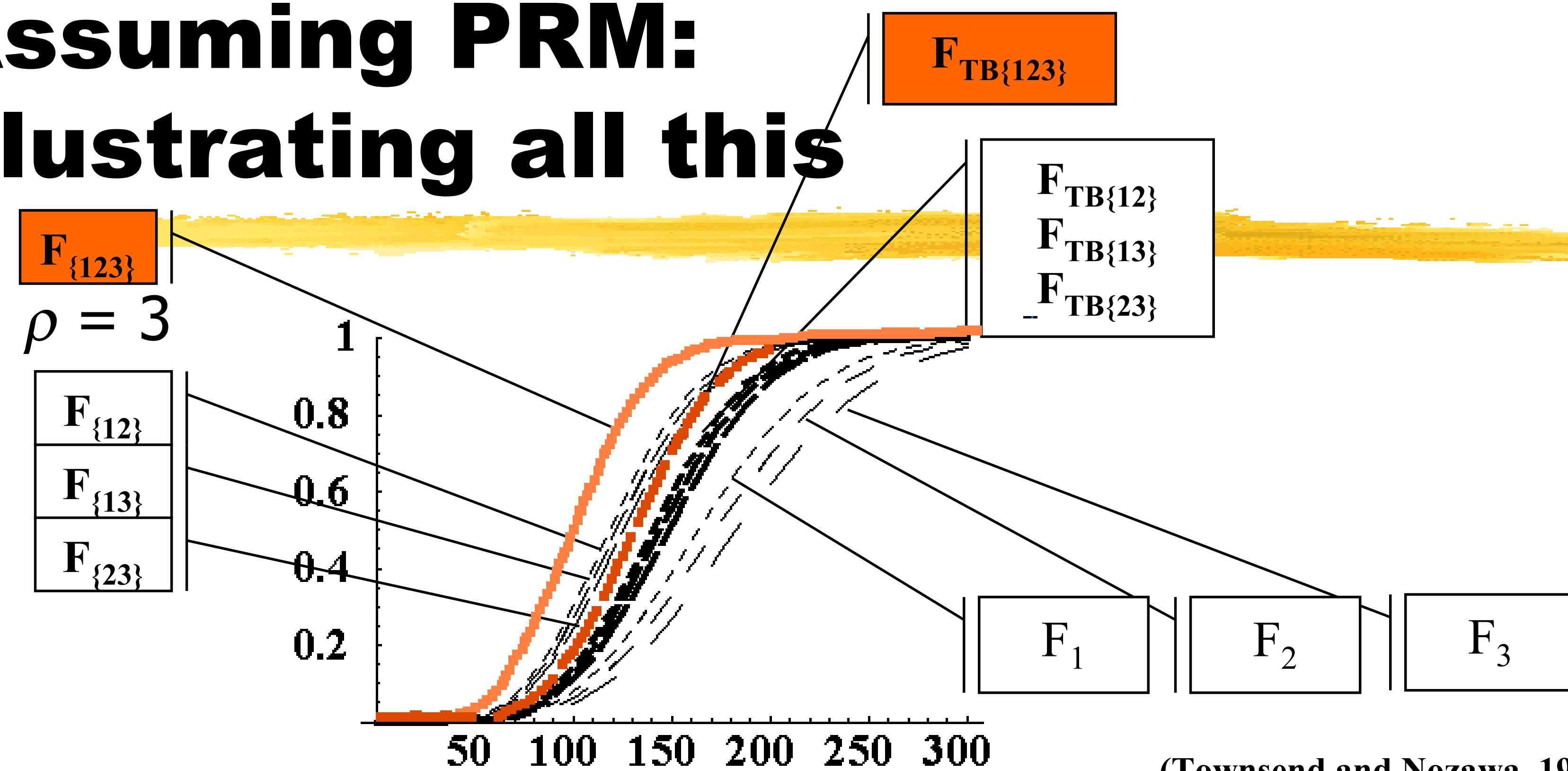
1. *#S* = 2

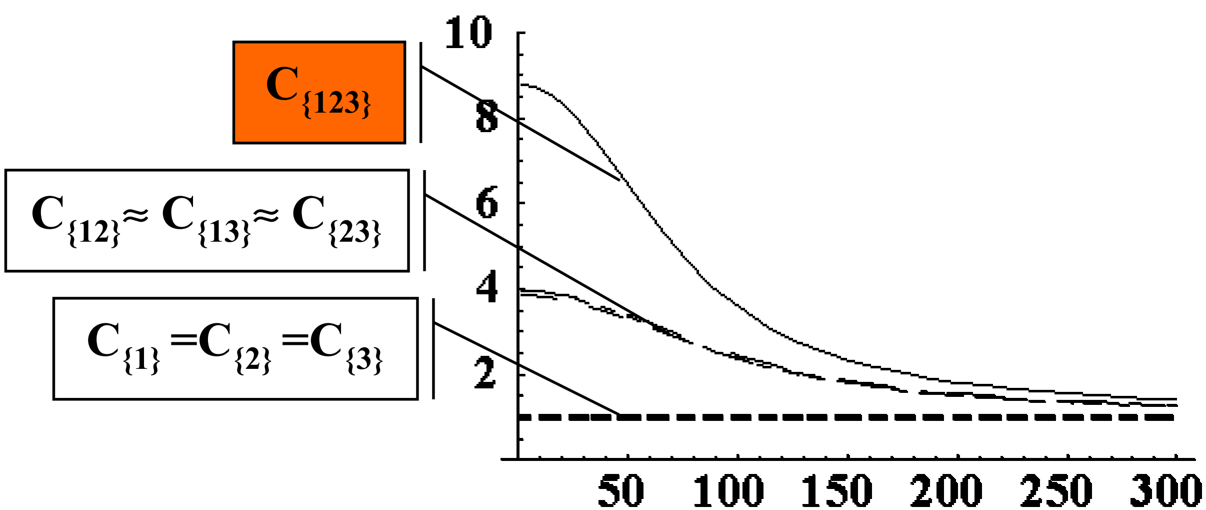


Capacity increase reflects an advantage in <u>processing time</u> above statistical facilitation.

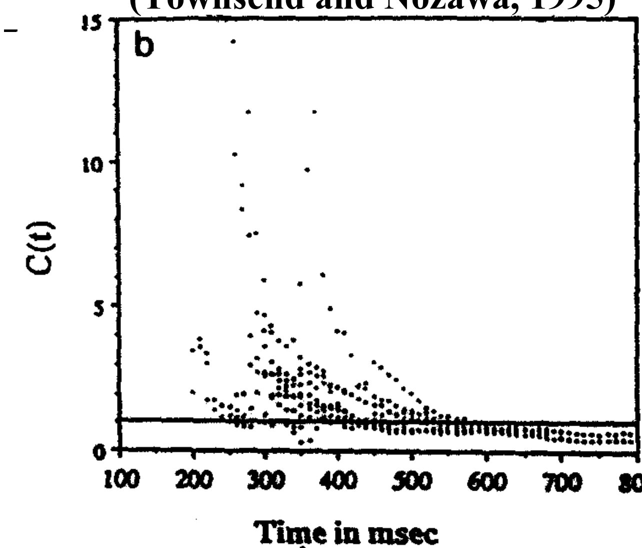
Assuming PRM: Illustrating all this

2. $\rho = 3$

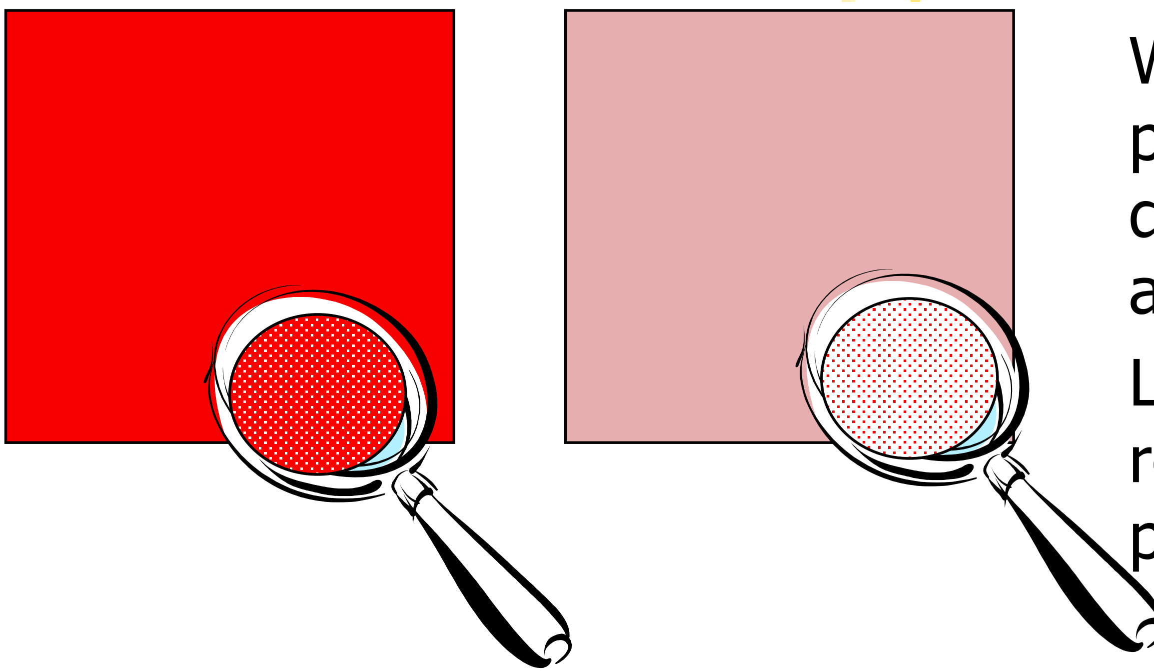




(Townsend and Nozawa, 1995)



An example?



What is the psychological difference between red and pink?

Luminance, that is, redundancy of the red photons reaching the retina.

Luminance := the increased capacity to perceive overall redness above the capacity of a single detector to perceive redness.

Conclusion

- In sum, density of information is coded (in time) even though there are no specific inputs that code luminance.
  - Thus, the PRM can learn to discriminate various luminance patches.
  - By fitting empirical capacity curves, it is possible to estimate k as a proportion of available racers per channel.
  - If $k >> \#S$, then there must be redundant channels in the system.

Thus, redundancy would no longer be an assumption.

- The redundancy conjecture (also see Logan):
  Can we develop tests to measure redundancy, and
  is redundancy present in many cognitive systems?

Thank you. This talk available at: **http://Prelude.psy.umontreal.ca/~cousined/home/talks.html**