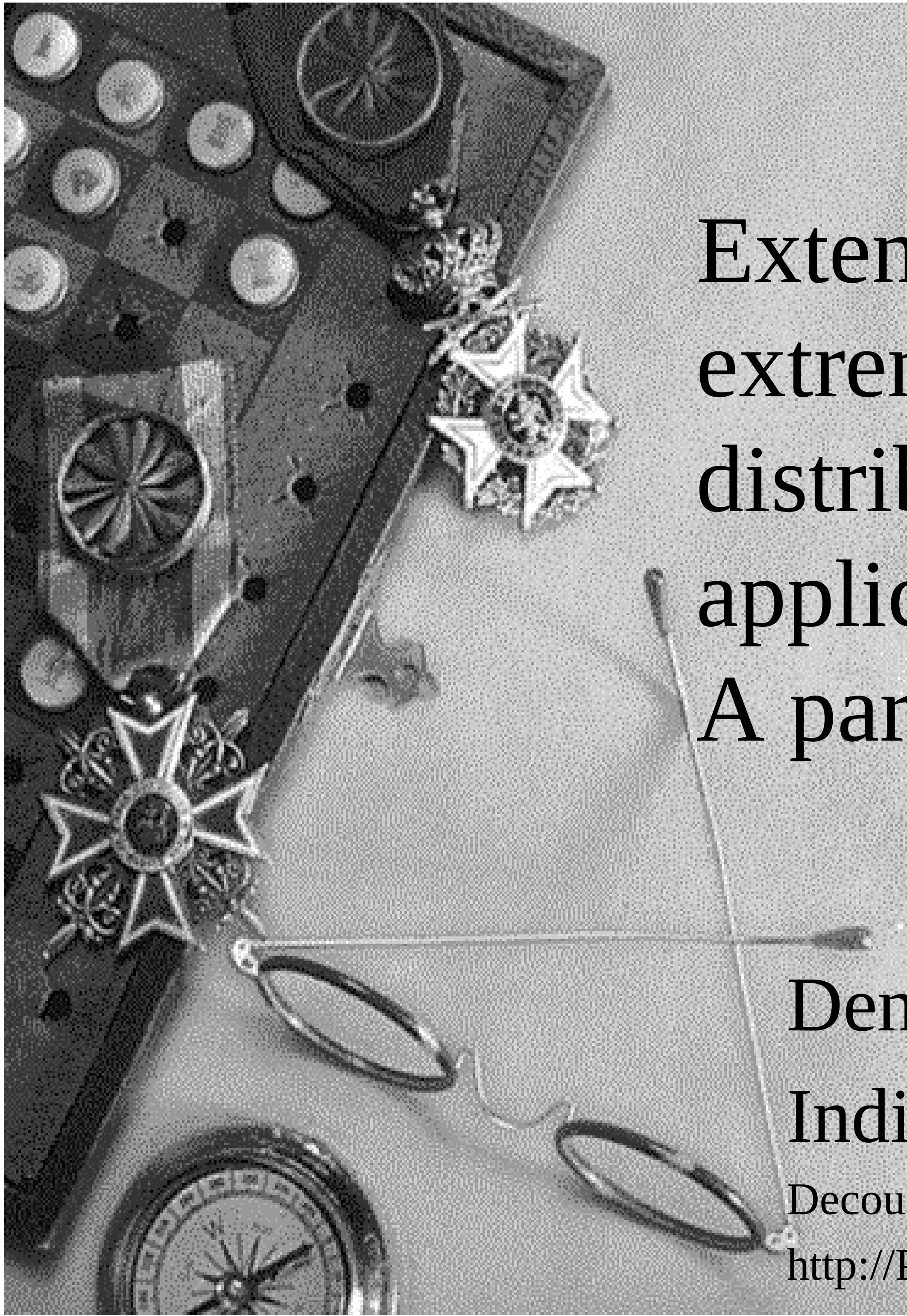

Extending statistics of extremes to non identical distributions and its application: A parallel race network

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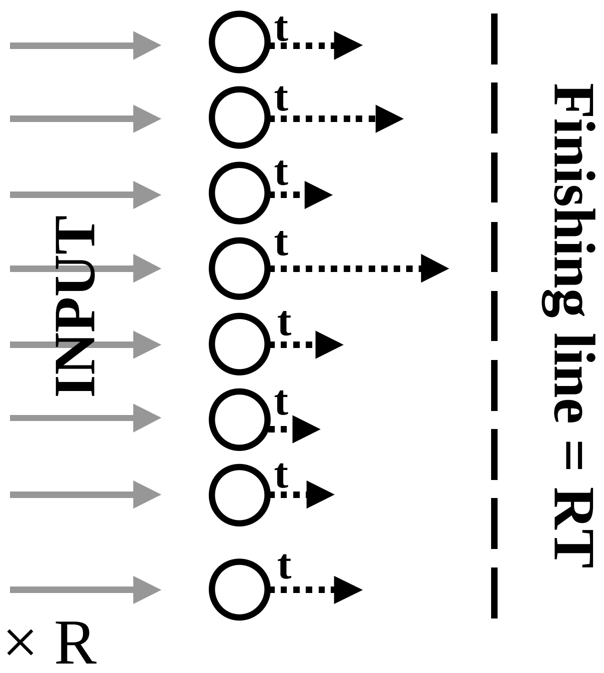
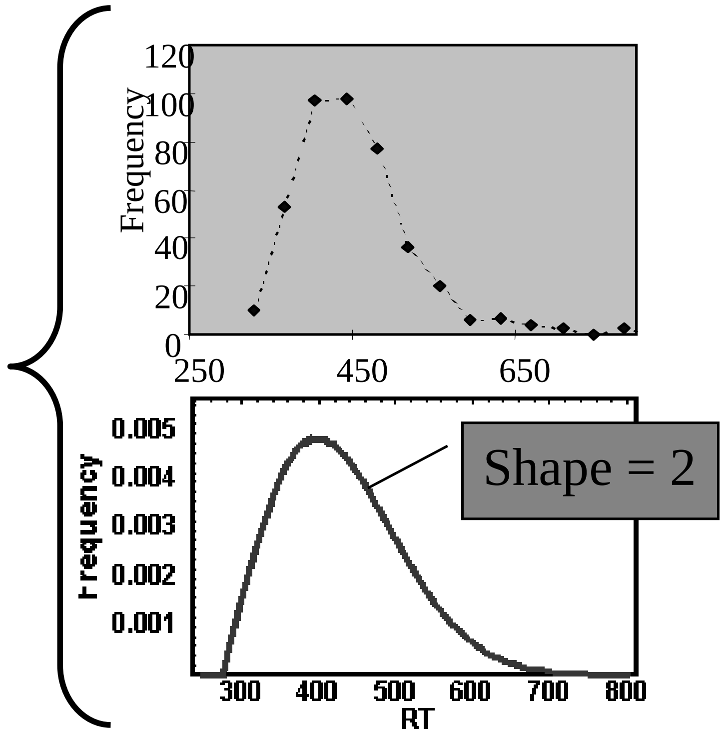
Parallel Race models: Overview

The fastest unit determines the reaction time (RT).





Examples:

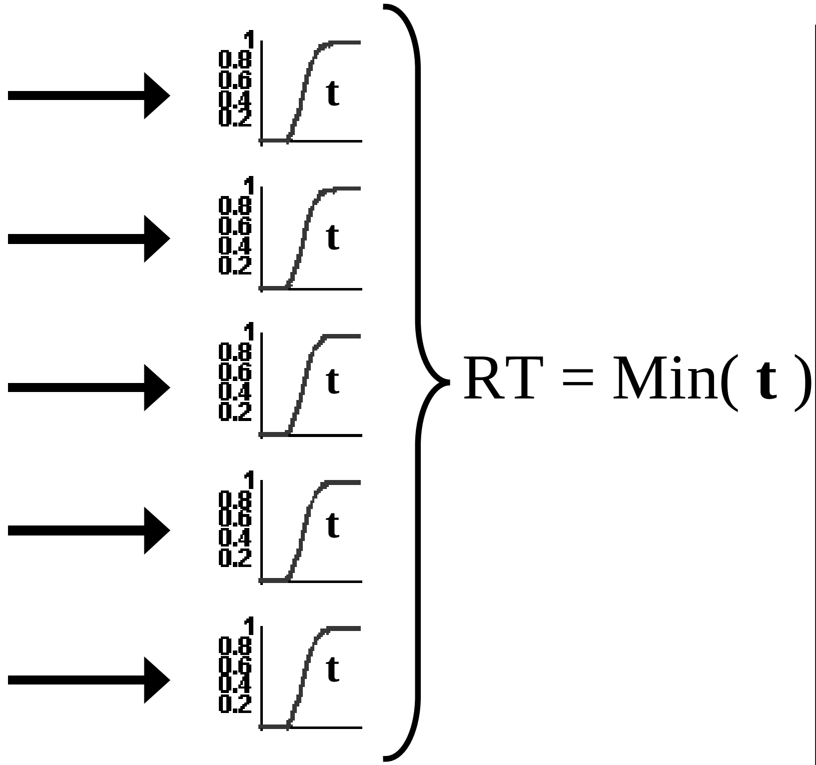
Logan (1988)

Bundesen (1990)

Predicts a Weibull distribution of RT.

(if R is large, i.e. > 50)

A- Theorems:
The criteria



Criteria:

- C_1: Independent
- C_2: Identical
- C_3: Lower-bounded
- C_4: Probabilities accumulate as a power curve (in the left tail at least).

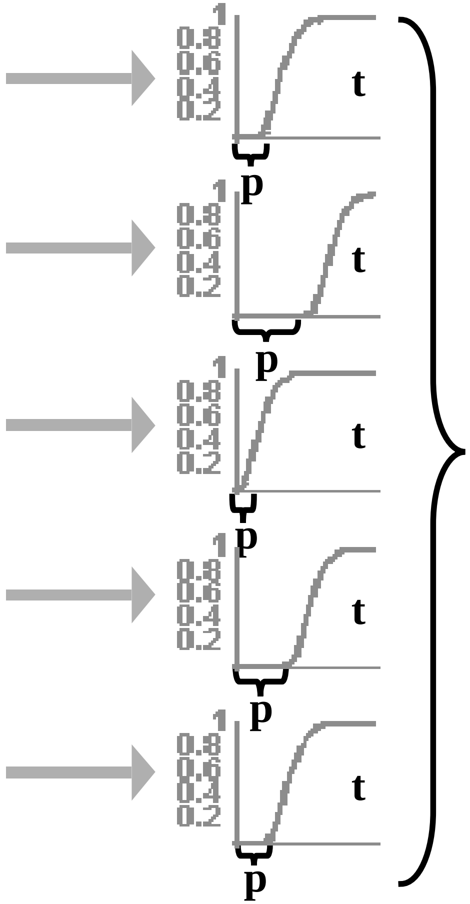
C_1 is not critical (Galambos, 1978)

C_2 is not psychologically plausible: Can we eliminate it?

C_3 & C_4 are technical assumptions that cannot be removed.

A- Theorems:
First case: Position is variable

As if the racers where not starting at the same time…



RT = Min($\mathbf{p} + \mathbf{t}$)

If $\mathbf{t} \sim \mathcal{F}$

$\mathbf{p} \sim \mathcal{G}$

Then

each unit $\sim \mathcal{F} * \mathcal{G}$

We proved that

if $\mathcal{F}$ satisfies C_3 & C_4

if $\mathcal{G}$ satisfies C_3 & C_4

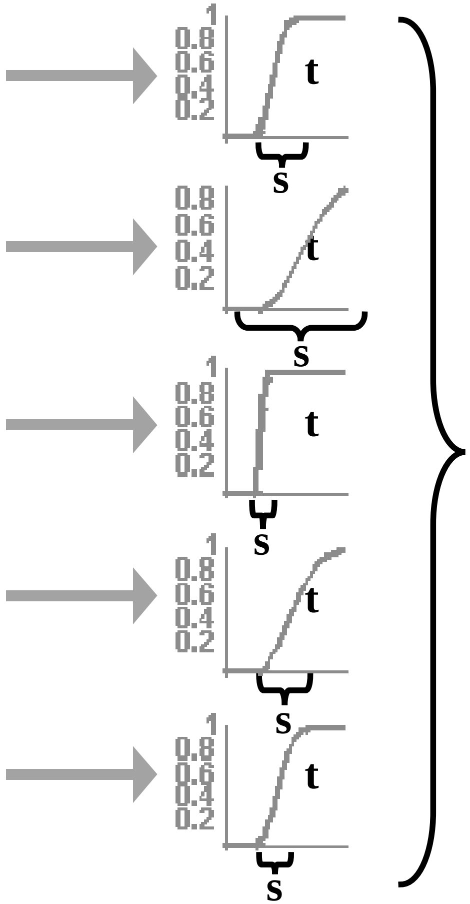
Then

$\mathcal{F} * \mathcal{G}$ does satisfy C_3 & C_4

Adding two factors still predicts a Weibull distribution for the winner.

A- Theorems:
Second case: Scale is variable

Each racer differs by their variance…



$$RT = \underset{s>0}{\mathrm{Min}}(\, \mathbf{s} \times \mathbf{t}\,)$$

If $\mathbf{t} \sim \mathcal{F}$

$\mathbf{s} \sim \mathcal{G}$

Then

each unit $\sim \mathcal{F} \wedge \mathcal{G}$

We proved that

if $\mathcal{F}$ satisfies C_3 & C_4

if $\mathcal{G}$ satisfies C_3 & C_4

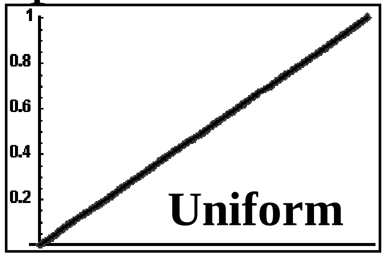
Then

$\mathcal{F} \wedge \mathcal{G}$ does satisfy C_3 & C_4

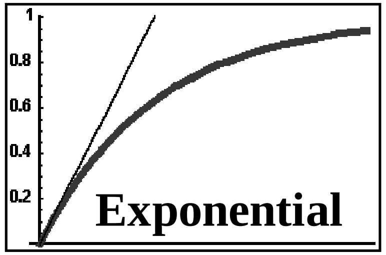
Multiply two factors still predicts a Weibull distribution for the winner.

A- Theorems: Can we predict the shape (skewness)?

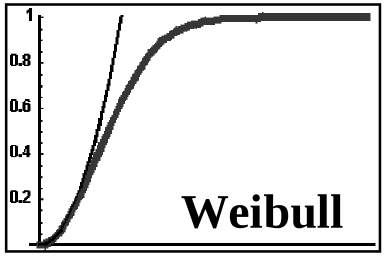
The shape γ of the distribution of the winner is given by the power curve fitting the left-hand tail:



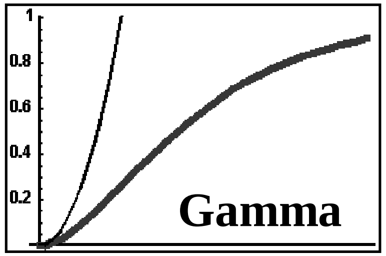
– The uniform is linear (x^1)



– So is the exponential in the left tail



– For the Weibull, shape can vary (here a quadratic tail, x^2)



– For the Gamma, the shape depends on the number of component (here, x^3).

The shape γ is given by the exponent.

A- Theorems: Shape of convolutions & productions

If

$\mathcal{F}$ has shape $\gamma_{\mathcal{F}}$

$\mathcal{G}$ has shape $\gamma_{\mathcal{G}}$

Then

$\mathcal{F} * \mathcal{G}$ has shape $\gamma_{\mathcal{F}} + \gamma_{\mathcal{G}}$

$\mathcal{F} \wedge \mathcal{G}$ has shape:

$\text{Min}(\gamma_{\mathcal{F}}, \gamma_{\mathcal{G}})$	if $0 \in \mathcal{F}$, $0 \in \mathcal{G}$
$\gamma_{\mathcal{F}}$	if $0 \in \mathcal{F}$ only
$\gamma_{\mathcal{F}} + \gamma_{\mathcal{G}}$	otherwise.

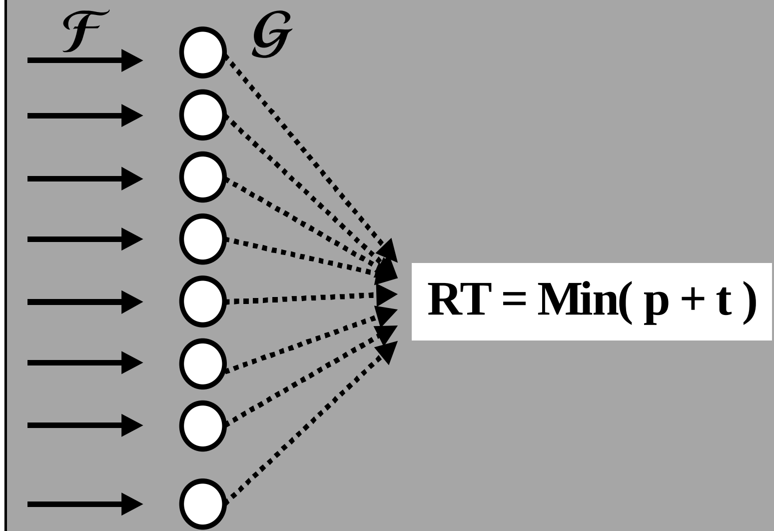
➔ the shape (the skewness) of the resulting distribution is not a free parameter.

Example: A 2-stage model.

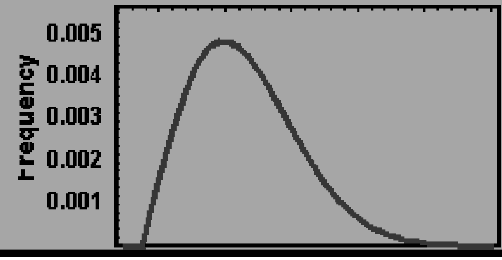
Stage 1: conduction with $\mathcal{F} \sim$ Uniform

Stage 2: decision with $\mathcal{G} \sim$ Exponential

The fastest decision is kept.



Since the shape of $\mathcal{F}$ is 1 and the shape of $\mathcal{G}$ is 1, the shape of $\mathcal{F} * \mathcal{G}$ is 2:



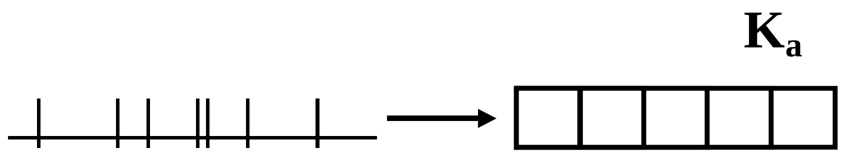
So far…

- The theorems shows that C_2 (identically distributed units) is not necessary to obtain a Weibull distribution, when Minima decides the RT.
- Skewness is not a free parameters (and empirically, $\gamma \approx 2$; Logan, 1992).

What general class of models could be based on a race? ➔Accumulator models

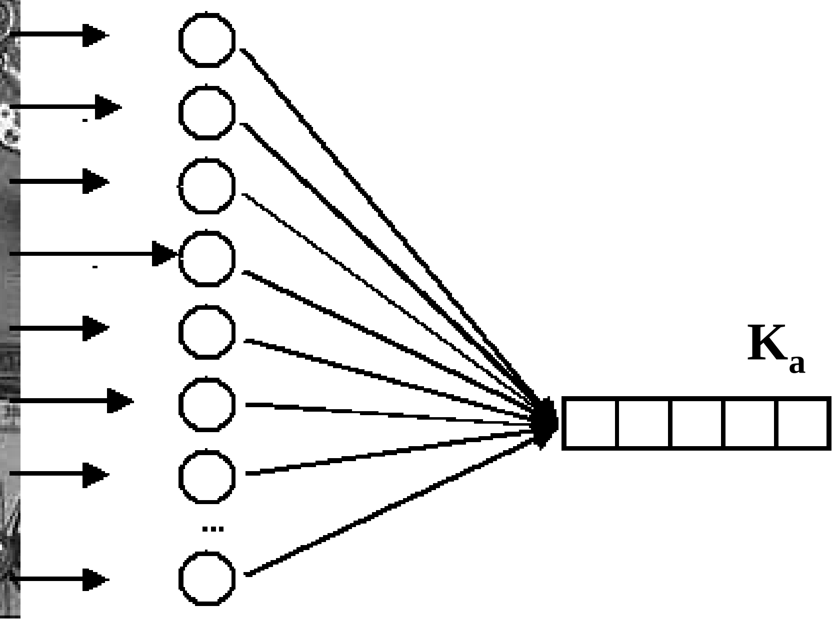
B- Class of models: Accumulator models

- Serial Poisson Race Model



$$\mathrm{RT} = \sum_{K_a} (t) \sim \mathit{Gamma}$$

- Parallel (Whatever*) Race Model



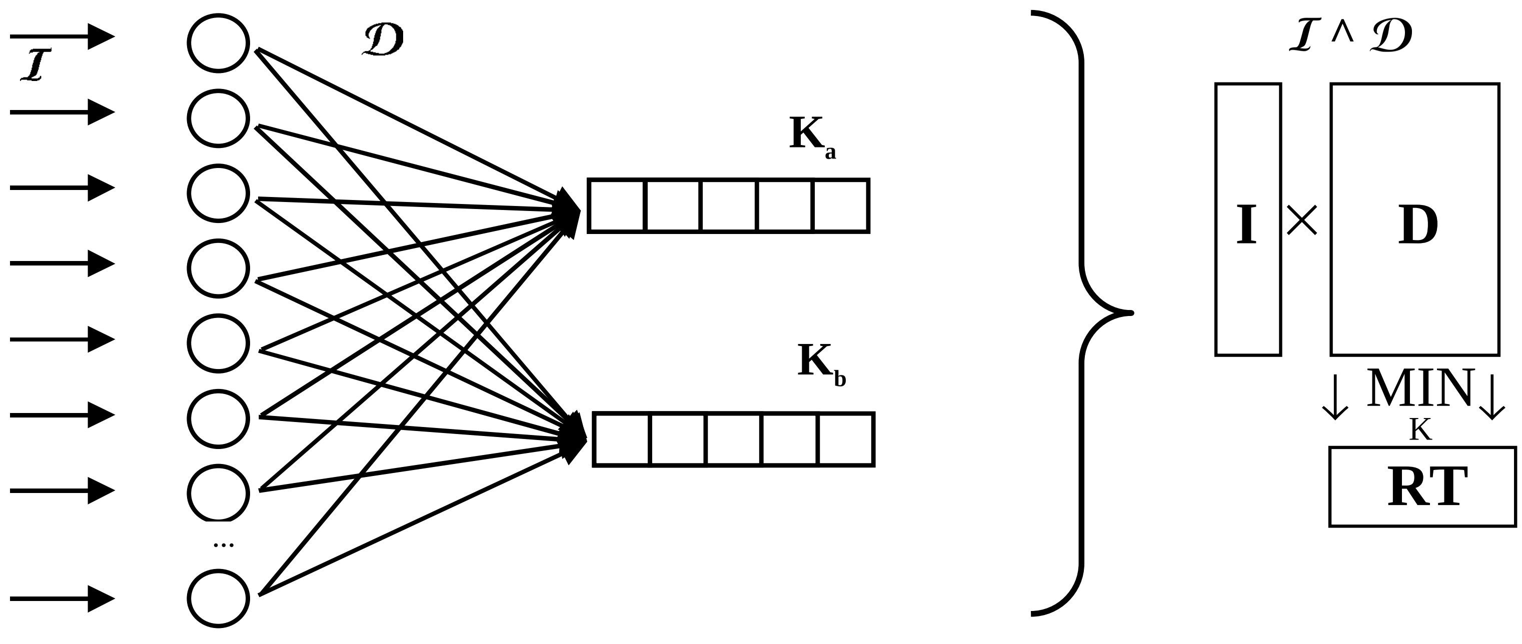
$$\mathrm{RT} = \underset{K_a}{Min}(t) \sim \mathit{Weibull}$$

because the same predictions apply if we wait for $\mathbf{K_a}$ winners instead of just one (Leadbetter, 1983)

*:as long as C_3 & C_4 are satisfied

B- Class of models: Accumulators with 2 alternatives

- In general, there are *n* alternatives, so we need *n* accumulators.
- A matrix representation is possible.



C- Application:
Parallel Race **Network**

Idea:

- K_i evidences are necessary for response i; Can K_i be learned?
- Information is conveyed through a connection matrix D introducing delays. Can the system learn to give priority to diagnostic information?

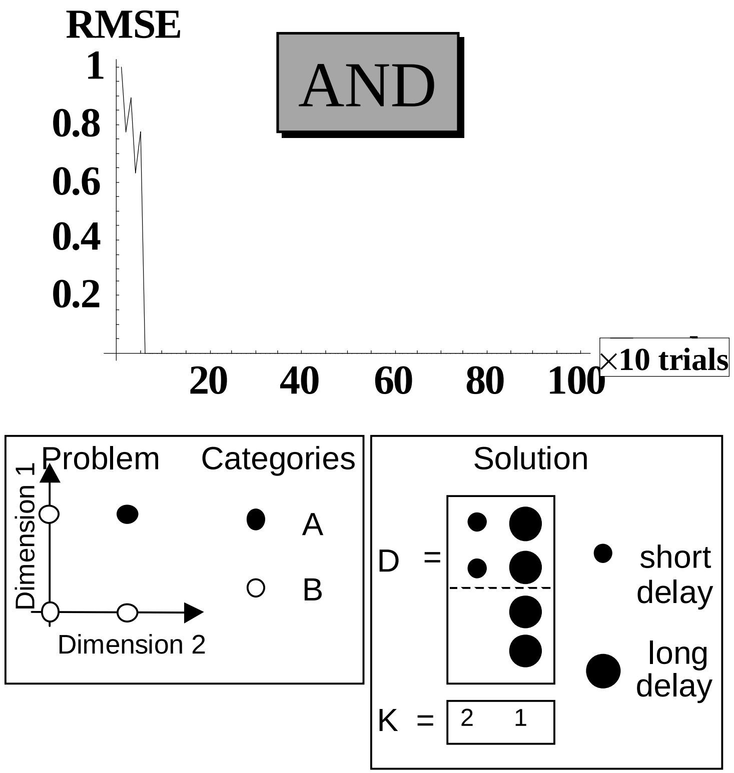
Answer:

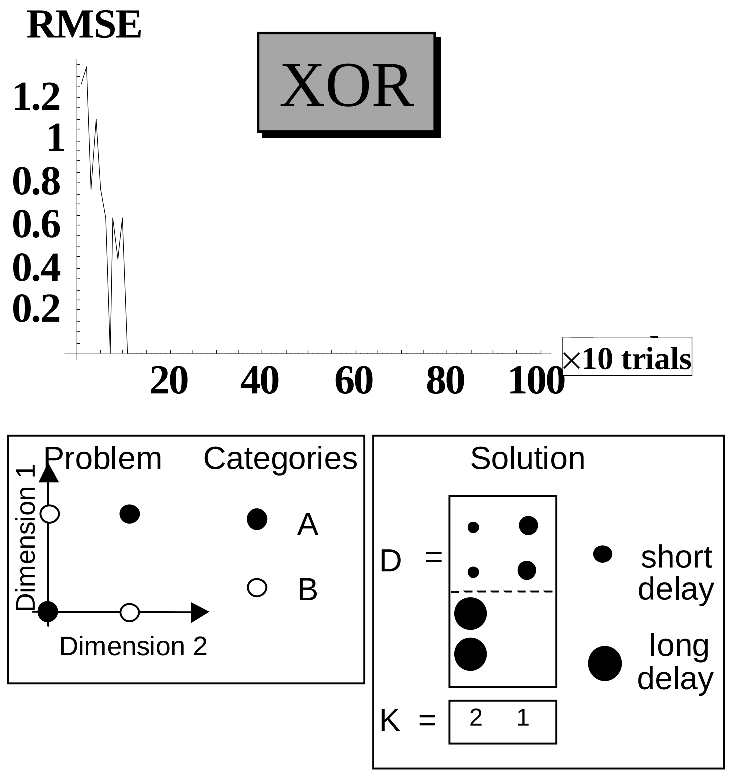
Yes, there exist a learning rule to adjust K_i and D.

C- Application: Learning rule

– If an output false alarmed (fired too soon), increase the delay of one connection;

– If an output missed (didn't fire), adjust the threshold K_a.





Conclusion

- The statistics of extremes are useful to infer the behavior of accumulator models;
- Integration of information in a “neural network” can be:
  - Σ as in Serial Poisson Race Model (sum) or standard connectionist models (weighted average)
  - Min (fastest units) as in the Parallel Race Model described here
  - Π (?) related to Cascade Models?