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# *A distributional test of exemplar race models*

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# *abstract*

*A prediction of the exemplar model proposed by Logan (1992) is that the shape parameter of the RT distribution should stay constant from session to session. One experiment involving a hybrid visual and memory search task shows that this is true for most of the subjects, including subjects in the varied mapping condition, who cannot associate responses to individual exemplars. Results of this experiment are therefore in contradiction with a race model based on exemplars. In a second experiment, subjects served in a categorical varied mapping condition in which the sets of stimuli serving as targets and distractors switched roles from trial to trial. The shape parameter was not constant for any of these subjects, although other measures (mean, standard deviation) were similar to results of the previous experiment.. Speed-accuracy trade-off is one factor that alters the fit of the distribution. It is suggested that tasks in which various levels of processing compete might prevent the shape parameter from being constant throughout practice.*

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# *Introduction*

A Weibull distribution of RT can result from a race where the fastest runner triggers the response. Assumptions needed are:

- 1) Every runner is a random variable
- 2) Runners are independent and identically distributed

Such a race model was used by Logan (1988, 1992) to implement his exemplar theory of automatization. This theory assumes:

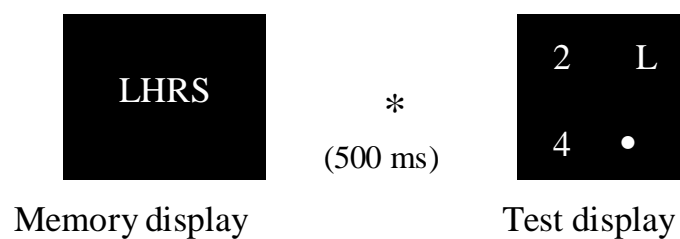
- 3) Obligatory encoding of every individual stimulus;
- 4) obligatory retrieval of all stored exemplars.

Using these assumptions, it is easy to show (see equation 1) that the distribution shape parameter  $c$  is not altered by practice.

With a transformation using quantiles, the probability density function (pdf) can be represented by lines of slope  $c$ .

According to exemplar race models, the "distribution lines" should be parallel from session to session. This **parallel line hypothesis** was tested in two experiments involving a hybrid memory and visual search task.

# Overview of the experiments



Stimuli were taken from four sets: {1, 4, 5, 8}, {2, 3, 6, 7}, {L, H, R, S} and {Z, B, G, F}. The Memory display and Test display were composed of 1, 2 or 4 characters. One session was composed of 864 trials. The learning phase lasted six sessions

# Experiment 1

16 subjects participated in one of the following two conditions. In the Consistent Mapping (**CM**) condition, the targets were chosen from one of the previous sets, while the distractors came from another as shown in Table 1. For half of the subjects, targets and distractors were all digits of all letters (homogeneous condition). For the other half (heterogeneous condition), there was a categorical distinction between targets and distractors. In the Varied Mapping (**VM**) condition, two sets were merged, targets and distractors being picked at random from this larger, homogeneous or heterogeneous, set.

Table 1: Targets and distractors assignment in CM

HOMOGENEOUS			HETEROGENEOUS		
<u>Ss</u>	<u>Targets</u>	<u>Distractor</u>	<u>Ss</u>	<u>Targets</u>	<u>Distractor</u>
S1	2, 3, 6, 7	1, 4, 5, 8	S5	2, 3, 6, 7	L, H, R, S
S2	1, 4, 5, 8	2, 3, 6, 7	S6	L, H, R, S	2, 3, 6, 7
S3	L, H, R, S	Z, B, G, F	S7	1, 4, 5, 8	Z, B, G, F
S4	Z, B, G, F	L, H, R, S	S8	Z, B, G, F	1, 4, 5, 8

## Results (1): Fit of the Weibull distribution

A  $\chi^2$  test of goodness of fit shows no significant difference between the empirical RT distribution and the Weibull distribution for **11** of the **16** subjects. Fits were good for 6 of the 8 Ss. in **CM**, and for 5 of the 8 Ss. in **VM**.

Figure 1 presents RT distributions for two typical subjects

Two of the five deviant subjects show highly significant **Speed-Accuracy Trade-Off (SATO)**.

A likelihood ratio test (LRT) was used to compare the quality of fit provided by the Weibull distribution against the fit of the Ex-Gaussian and the Log-Normal distributions. The Weibull distribution provided the best fit for every session of every subject without exception.

## Results (2): Parallel line hypothesis

A likelihood ratio test (LRT) was used to compare the fit when the  $c$  parameter was free to vary against a model where the  $c$  parameter was constrained to be equal throughout the 6 sessions.

Figure 2 presents the  $c$  parameters for two typical subjects

For 5 subjects, the  $c$  parameters changed significantly from session to session. These are **the same** five subjects whose data did not fit a Weibull distribution.

# Experiment 2

8 subjects participated in this experiment. The stimuli were taken from the same sets as before. However, their role as targets or distractors was decided at random for every trial, as shown in Table 2. This condition is termed **Categorical Varied Mapping (CVM)** because the stimulus-response mapping varied over trials. However, since the stimuli of any given set served only as targets or distractors on any given trial, subjects could develop a categorical knowledge of the sets. This experiment was identical to Experiment 1 in all other aspects.

Table 2: Targets and distractors assignment in Exp. 2

HOMOGENEOUS			HETEROGENEOUS		
Ss	Targets	Distractors	Ss	Targets	Distractors
S1&	1, 4, 5, 8	2, 3, 6, 7	S5&	1, 4, 5, 8	Z, B, G, F
S2	2, 3, 6, 7	1, 4, 5, 8	S6	Z, B, G, F	1, 4, 5, 8
S3&	L, H, R, S	Z, B, G, F	S7&	1, 4, 5, 8	Z, B, G, F
S4	Z, B, G, F	L, H, R, S	S8	Z, B, G, F	1, 4, 5, 8

The stimuli are presented on two lines, preceded by a {.  
Two subjects had the same stimuli.

## Results (1): Fit of the Weibull distribution

RT distributions did not differ from a Weibull distribution for **6** of the **8** subjects (exceptions are Ss 3 and 5). Figure 3 presents two typical subjects.

Subject 5 exhibited significant SATO. We cannot explain at present why subject 3's RTs did not fit a Weibull distribution.

## Results (2): Parallel line hypothesis

No single subject had a constant shape parameter throughout learning (all  $\chi^2 > 16.2$ ,  $p < .006$ ). See figure 4.

This is a very surprising result, because on all other aspects (mean RT, slope as a function of set sizes, power learning curves), the performance observed in the **CVM-HETERO** condition resembled that obtained in the **CM** condition of Exp 1, while the performance observed in **CVM-HOMO** was almost identical to that obtained in **VM** of Exp 1.

However, the distribution analyses revealed that **CVM** subjects behaved differently from all other groups.



# Conclusion

## Exemplar race models

Exemplar theories cannot account for these results since the response associated to each exemplar varies from trial to trial in both **VM** and in **CVM** conditions. Therefore, access to prior exemplar is uninformative. If a race model is to account for the **VM** results, the runners have to be at a lower level than the exemplars.

## Weibull distribution

a) The basic assumption of the Weibull distribution is the independence of the processes participating in the decision. Between-trial dependencies (such as Speed-Accuracy Trade-Off) violate this assumption. If the SATO is large, it disrupts the RT distribution.

b) The Weibull adequately explains the CM results in terms of a race among exemplars. However, other interpretations are possible. In **CM**, exemplars are mapped onto sets (targets vs. distractors) which are also consistently associated to responses. In **VM** condition, neither exemplars nor sets are consistently mapped to responses. The only race possible would be among the exemplar features.

One possible though post hoc explanation supposes that the race can occur at different levels. In **CM** and **VM** conditions, the same level of response would always be used to make the decision. In **CVM**, however, the same level would not systematically win the race. This instability might alter the RT distributions (see figure 5).

## Equation 1

If the RT distribution for one exemplar is given by the Weibull distribution (assumptions 1 and 2) then, as the number of exemplars  $n$  increases (assumptions 3 and 4),

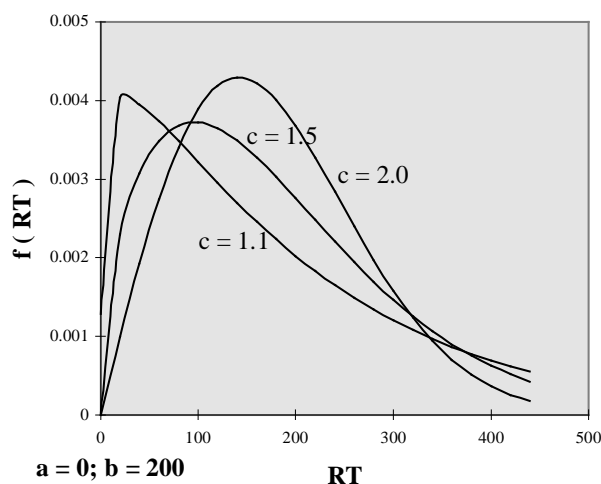
$$\begin{aligned}
 F_n(RT) &= 1 - (1 - F(RT))^n \\
 &= 1 - \left( e^{-\left(\frac{RT-a}{b}\right)^c} \right)^n \\
 &= 1 - e^{-n\left(\frac{RT-a}{b}\right)^c} \\
 &= 1 - e^{-\left(\frac{1}{n^{-1/c}}\right)^c \left(\frac{RT-a}{b}\right)^c} = 1 - e^{-\left(\frac{RT-a}{bn^{-1/c}}\right)^c}
 \end{aligned}$$

The resulting distribution is also a Weibull distribution, with the scale parameter  $b$  reduced. However,  $c$  is not altered by practice.

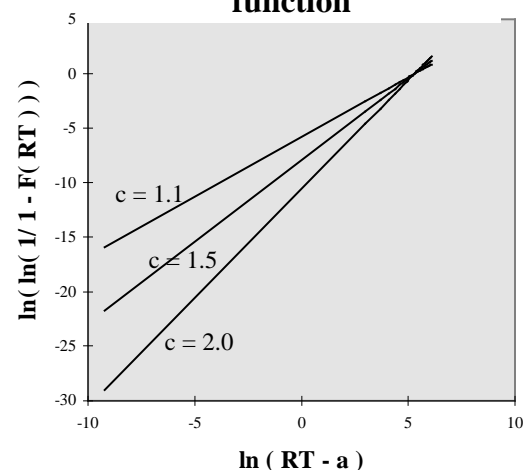
The following illustrates the linearization of the

PDF:

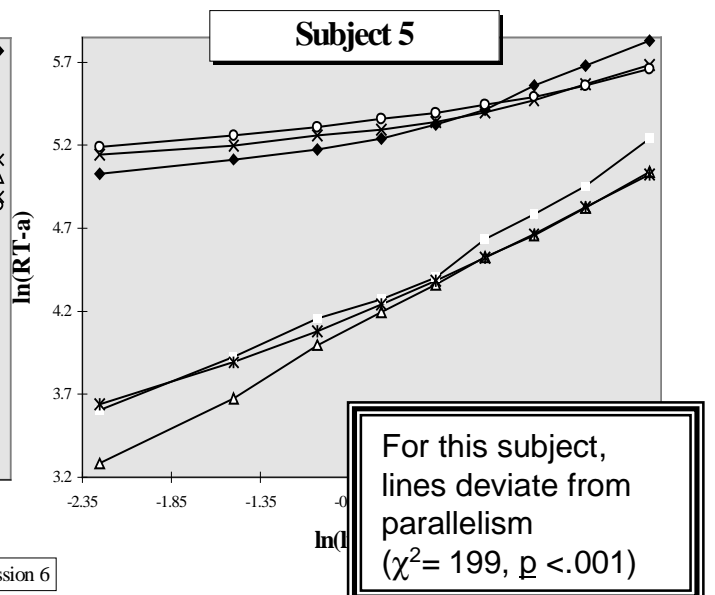
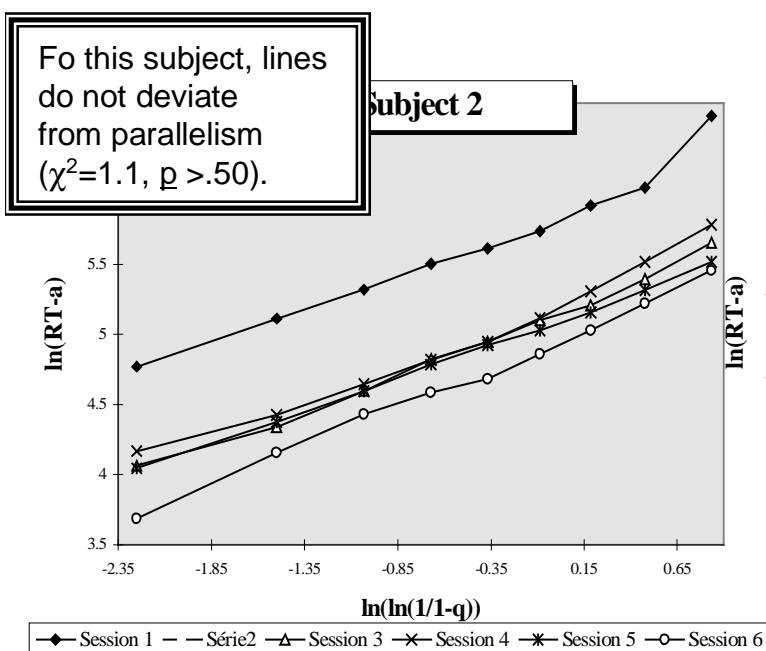
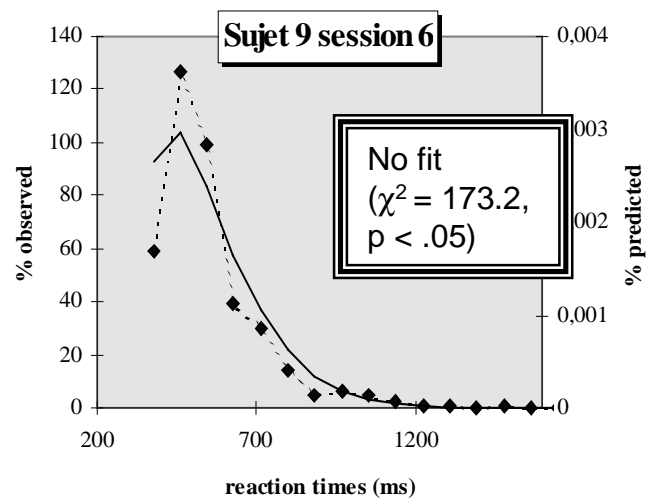
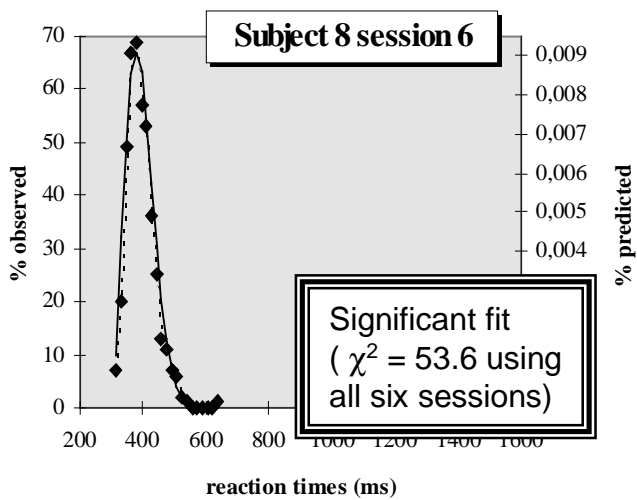
**Probability density function**



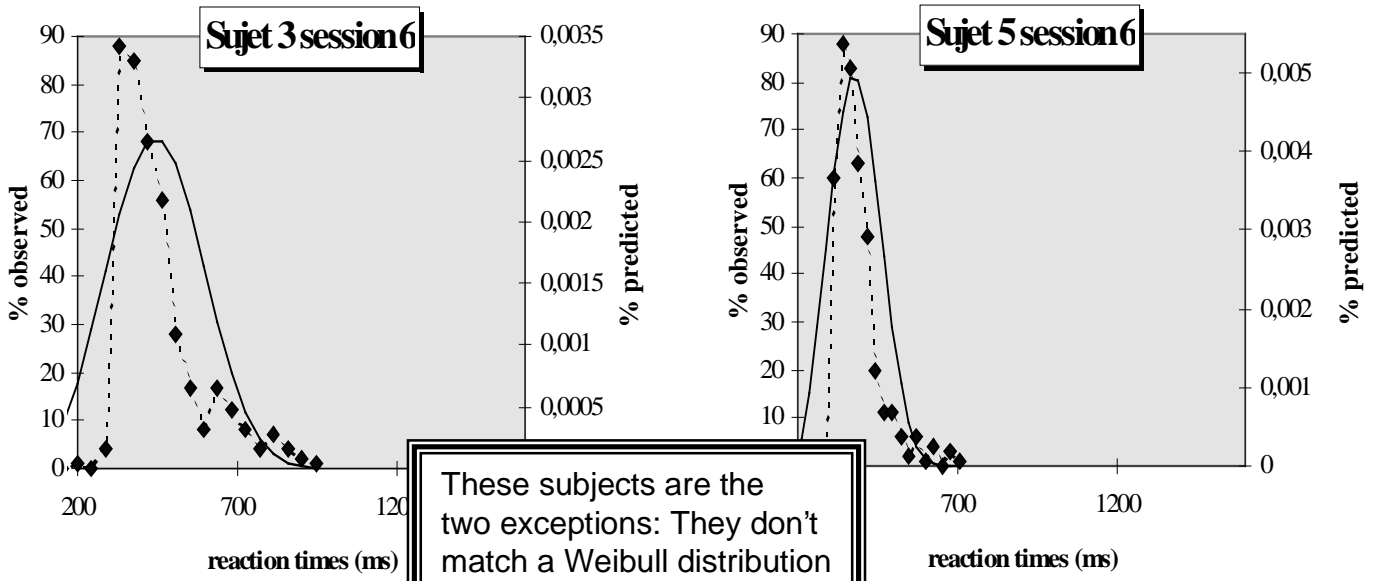
**Linearized cumulative prob. function**



# Figures 1 and 2



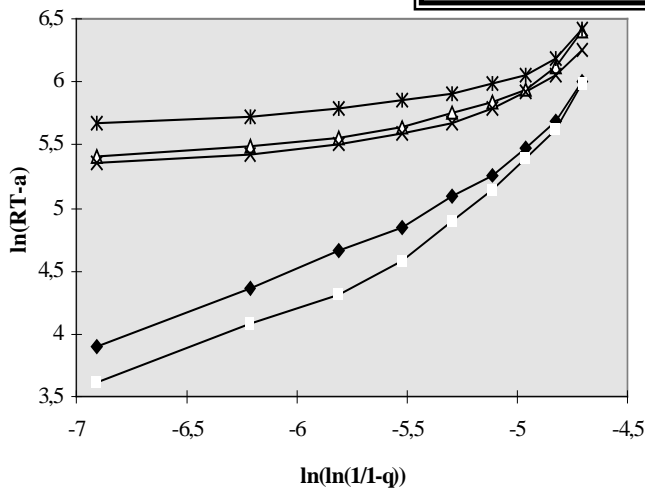
# Figures 3 and 4



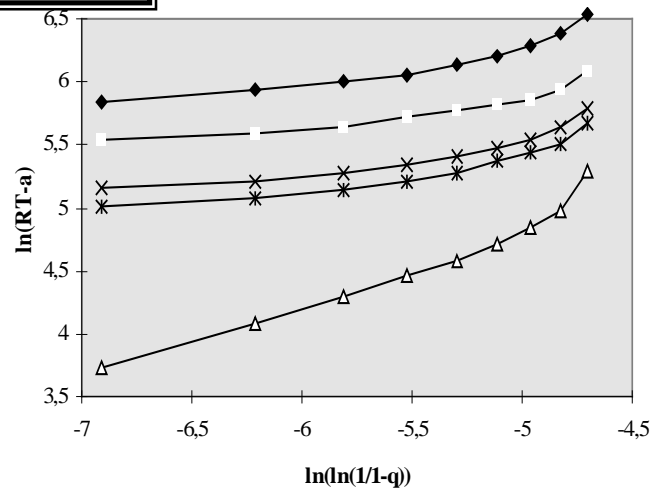
These subjects are the two exceptions: They don't match a Weibull distribution ( $\chi^2 > 224,3$  for 6 sessions) nor are their lines parallel ( $\chi^2 > 400$ )

Subject 3

Subject 5

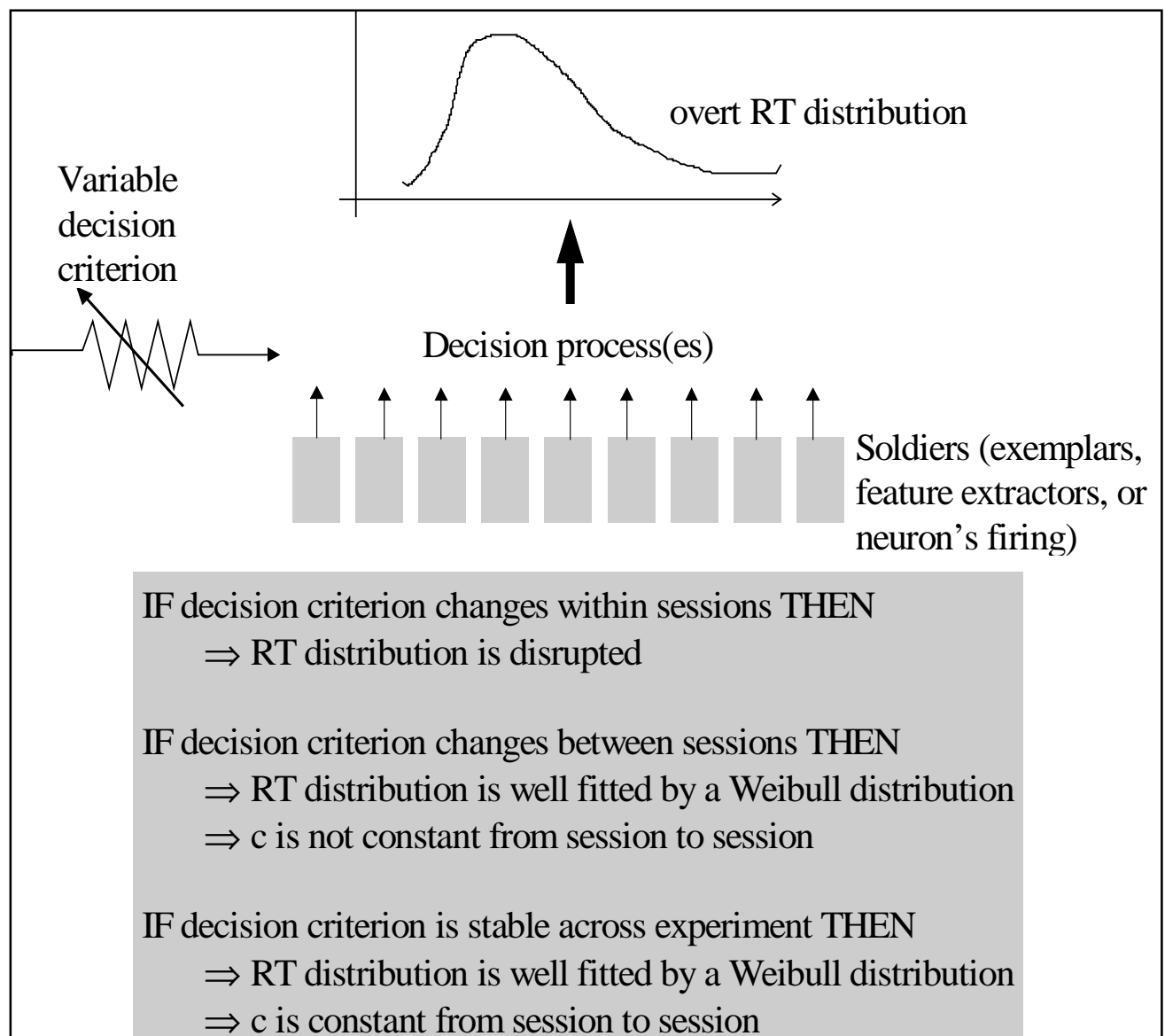


◆ session 1 — session 3 ▲ session 4 × session 5 \* session 6



◆ session 1 — session 3 ▲ session 4 × session 5 \* session 6

# Figure 5: a model of RT distribution



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***C, the shape parameter of the RT distribution, is represented by a line of slope  $c$  using adequate transformation (Weibull, 1951).***

***According to exemplar race models, distribution lines should be parallel because the number of exemplars  $n$  is a linear function of practice  $N$ .***